Fill Ups of Sequences and Series

Q.1. The sum of integers from 1 to 100 that are divisible by 2 or 5 is .... (1984 - 2 Marks)

Ans. 3050

Sol. The sum of integers from 1 to 100 that are divisible by 2 or = sum of integers from 1 to 100 divisible by 2 + sum of integers from 1 to 100 divisible by 5 – sum of integers from 1 to 100 divisible by 10

= (2 + 4 + 6 +...+ 100) + (5 + 10 + 15 +...+ 100)

– (10 + 20 +...+ 100)

$$=\frac{50}{2}[2\times2+49\times2]+\frac{20}{2}[2\times5+19\times5]$$

$$=\frac{10}{2}[2\times10+9\times10]=$$

= 2550 + 1050 – 550 = 3050

Q.2. The solution of the equation $\log_7 \log_5(\sqrt{x+5}+\sqrt{x})$ **is .... (1986 - 2 Marks)**

Ans. 4

Sol. The given equation is

$$\log_7 \log_5 (\sqrt{x+5}+\sqrt{x})=0$$

$$\Rightarrow \log_5 (\sqrt{x+5}+\sqrt{x})=1$$

Squaring both sides

$$\Rightarrow \sqrt{x+5}+\sqrt{x}=5 \Rightarrow \sqrt{x+5}=5-\sqrt{x} \cdot 10\sqrt{x}+x \Rightarrow 10\sqrt{x}=20$$

$$\Rightarrow \sqrt{x}=2 \Rightarrow x=4$$

Q.3. The sum of the first n terms of the series

$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots\ldots\ldots$ **is**

n (n + 1) ^{2}/2, when n is even. When n is odd, the sum is .... (1988 - 2 Marks)

Ans. $\frac{n^2(n+1)}{2}$

Sol. When n is odd, let n = 2m + 1

$\therefore$ The req. sum

$=1^2+2.2^2+3^2+2.4^2+\ldots+2(2m)^2+(2m+1)^2$

$=\Sigma(2m+1)^2+4[1^2+2^2+3^2+\ldots+m^2]$

$=\frac{(2m+1)(2m+2)(4m+2+1)}{6}+\frac{4m(m+1)(2m+1)}{6}$

$\frac{(2m+1)(m+1)}{6}$ [2 (4m + 3) + 4m]

$=\frac{(2m+1)(2m+2)(6m+3)}{6}=\frac{(2m+1)^2(2m+2)}{2}$

$=\frac{n^2(n+1)}{2}$ $[\because 2m+1=n)$

Q.4. Let the harmonic mean and geometric mean of two positive numbers be the ratio 4 : 5. Then the two number are in the ratio .... (1992 - 2 Marks)

Ans. 4 : 1 or 1 : 4

Sol. Let a and b be two positive numbers.

Then, H.M. $=\frac{2ab}{a+b}$ and G.M. $=\sqrt{ab}$

ATQ HM : GM = 4 : 5

$\therefore \frac{2ab}{(a+b)\sqrt{ab}}=\frac{4}{5}$

$\Rightarrow \frac{2\sqrt{ab}}{a+b}=\frac{4}{5} \Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{5+4}{5-4}$

$$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{9}{1} \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = 3, -3$$

$$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{3+1}{3-1}, \frac{-3+1}{-3-1} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = 2, \frac{1}{2} \Rightarrow \frac{a}{b} = 4, \frac{1}{4}$$

a : b = 4 : 1 or 1 : 4

Q.5. For any odd integer n ≥ 1, $n^3-(n-1)^3+...+(-1)^{n-1}1^3$ = ... (1996 - 1 Mark)

Sol. Since n is an odd integer, $(-1)^{n-1} = 1$ and n – 1, n – 3, n – 5, ... are even integers.

We have

$$n^3-(n-1)^3+(n-2)^3-(n-3)^3+.....+(-1)^{n-1}1^3$$

$$= n^3+(n-1)^3+(n-2)^3+......+1^3-2\,[(n-1)^3$$

$$+(n-3)^3+.....+2^3]$$

$$= [n^3+(n-1)^3+(n-2)^3+......+1^3]$$

$$-2\times 2^3\left[\left(\frac{n-1}{2}\right)^3+\left(\frac{n-3}{2}\right)^3+......+1^3\right]$$

[∴ n – 1, n – 3.........are even integers] Here the first square bracket contain the sum of cubes of 1st n natural numbers. Whereas the second square bracket contains the sum of the cubes of natural numbers from 1 to $\left(\frac{n-1}{2}\right)$, where n – 1, n – 3, ... are even integers. Using the formula for sum of cubes of 1st n natural numbers we get the summation

$$=\left[\frac{n(n+1)}{2}\right]^2-16\left[\left\{\frac{1}{2}\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)\right\}\right]^2$$

$$=\frac{1}{4}n^2(n+1)^2-16\,\frac{(n-1)^2(n+1)^2}{16\times 4}$$

$$=\frac{1}{4}(n+1)^2[n^2-(n-1)^2]=\frac{1}{4}(n+1)^2(2n-1)$$

Q.6. Let p and q be roots of the equation $x^2 – 2x + A = 0$ and let r and s be the roots of the equation $x^2 – 18x + B = 0$. If $p < q < r < s$ are in arithmetic progression, then A = ... and B = ... (1997 - 2 Marks)

Ans. –3, 77

Sol. It is given $p + q = 2$, $pq = A$ and $r + s = 18$, $rs = B$ and it is given that p,q, r, are in A.P.

Therefore, let $p = a – 3s$, $q = a – d$, $r = a + d$ and $s = a + 3d$.

As $p < q < r < s$, we have $d > 0$

Now, $2 = p + q = a – 3d + a – d = 2a – 4d$

$\Rightarrow a – 2d = 1$ ....(1)

Again $18 = r + s = a + d + a + 3d$

$\Rightarrow 18 = 2a + 4d \Rightarrow 9 = a + 2d$. ....(2)

Subtracting (1) from (2) $\Rightarrow 8 = 4d \Rightarrow 2 = d$

Putting in (2) we obtain $a = 5$

$\therefore p = a – 3d = 5 – 6 = – 1$,

$q = a – d = 5 – 2 = 3$ $r = a + d = 5 + 2 = 7$,

$s = a + 3d = 5 + 6 = 11$

Therefore, $A = pq = – 3$ and $B = rs = 77$.

Match the following of Sequences and Series

PASSAGE - 1

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r – 1).

Let $T_r = V_{r+1} - V_{r-2}$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2, ...

1. The sum $V_1 + V_2 + ... + V_n$ is (2007 -4 marks)

(a) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$

(b) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(c) $\frac{1}{2}n(2n^2 - n + 1)$

(d) $\frac{1}{3}(2n^3 - 2n + 3)$

Ans. Sol.

(b) $V_1 + V_2 + ... + V_n = \sum_{r=1}^{n} V_r = \sum_{r=1}^{n}\left(r^3 - \frac{r^2}{2} + \frac{r}{2}\right)$

$= \Sigma n^3 - \frac{\Sigma n^2}{2} + \frac{\Sigma n}{2}$

$= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$

$= \frac{n(n+1)}{4}\left[n(n+1) - \frac{2n+1}{3} + 1\right]$

$= \frac{n(n+1)(3n^2 + n + 2)}{12}$

2. Tr is always (2007 -4 marks)

(a) an odd number

(b) an even number

(c) a prime number

(d) a composite number

Ans. Sol.

(d) $T_r = V_{r+1} - V_{r-2}$

$$= \left[(r+1)^3 - \frac{(r+1)^2}{2} + \frac{r+1}{2}\right] - \left[r^3 - \frac{r^2}{2} + \frac{r}{2}\right] - 2$$

$= 3r^2 + 2r + 1$

$T_r = (r + 1)(3r – 1)$

For each r, T_r has two different factors other than 1 and itself.

∴ T_r is always a composite number.

3. Which one of the following is a correct statement ? (2007 -4 marks)

(a) Q_1, Q_2, Q_3, ... are in A.P. with common difference 5
(b) Q_1, Q_2, Q_3, ... are in A.P. with common difference 6
(c) Q_1, Q_2, Q_3, ... are in A.P. with common difference 11
(d) $Q_1 = Q_2 = Q_3 = ...$

Ans. Sol.

(b) ∴ $Q_{r+1} - Q_r = T_{r+2} - T_{r+1} - (T_{r+1} - T_r)$ $= T_{r+2} - 2T_{r+1} + Tr$

$= (r + 3)(3r + 5) - 2(r + 2)(3r + 2) + (r + 1)(3r - 1)$

∵$Q_{r+1} – Q_r = 6(r + 1) + 5 – 6r – 5 = 6$ (constant)

∴ ∵1, Q_2, Q_3, ... are in AP with common difference 6.

PASSAGE -2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

4. Which one of the following statements is correct ? (a) $G_1 > G_2 > G_3 > \ldots$ (2007 -4 marks)

(b) $G_1 < G_2 < G_3 < \ldots$ (c) $G_1 = G_2 = G_3 = \ldots$ (d) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots$.

Ans. Sol.

(c) Given $A_1 = \frac{a+b}{2}, G_1 = \sqrt{ab},\ H_1 = \frac{2ab}{a+b}$

also $A_n = \frac{A_{n-1} + H_{n-1}}{2}, G_n = \sqrt{A_{n-1}H_{n-1}}$

$H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$

$\Rightarrow G_n^2 = A_n H_n \Rightarrow A_n H_n = A_{n-1} H_{n-1}$

Similarly we can prove

$A_nH_n = A_{n-1}H_{n-1} = A_{n-2}H_{n-2} = \ldots = A_1H_1$

$\Rightarrow A_nH_n = ab$

$\therefore\ G_1^2 = G_2^2 = G_3^2 \ldots = ab$

$\Rightarrow G_1 = G_2 = G_3 \ldots \sqrt{ab}$

5. Which one of the following statements is correct ? (a) $A_1 > A_2 > A_3 > \ldots$ (2007 -4 marks)

(b) $A_1 < A_2 < A_3 < \ldots$ (c) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$ (d) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

Ans. Sol.

(a) We have

$$A_n = \frac{A_{n-1} + H_{n-1}}{2}$$

$$\therefore \quad A_n - A_{n-1} = \frac{A_{n-1} + H_{n-1}}{2} - A_{n-1}$$

$$= \frac{H_{n-1} - A_{n-1}}{2} < 0 \ (\because A_{n-1} > H_{n-1})$$

$\Rightarrow A_n < A_{n-1}$ or $A_{n-1} > A_n$ $\therefore$ We can conclude that $A_1 > A_2 > A_3 > ....$

6. Which one of the following statements is correct ? (a) $H_1 > H_2 > H_3 > ...$ (2007 -4 marks)

(b) $H_1 < H_2 < H_3 < ...$ (c) $H_1 > H_3 > H_5 > ...$ and $H_2 < H_4 < H_6 < ...$ (d) $H_1 < H_3 < H_6 < ...$ and $H_2 > H_4 > H_6 > ...$

Ans. Sol.

(b) We have $A_n H_n = ab \Rightarrow H_n = \frac{ab}{A_n}$

$$\because \quad \frac{1}{A_{n-1}} < \frac{1}{A_n} \Rightarrow H_{n-1} < H_n \quad \therefore H_1 < H_2 < H_3 < ....$$

Subjective questions of Sequences and Series

Q.1. The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation. $2A + G^2 = 27$

Find the two numbers. (1979)

Ans. 3 and 6 or 6 and 3

Sol. Let the two numbers be a and b, then

$$\frac{2ab}{a+b} = 4 \ldots(1); \quad \frac{a+b}{2} = A; \sqrt{ab} = G$$

Also $2A + G^2 = 27 \Rightarrow a + b + ab = 27$ ....(2)

Putting ab = 27 – (a+b) in eqn. (1), we get

$$\frac{54 - 2(a+b)}{a+b} = 4 \Rightarrow a + b = 9 \text{ then } ab = 27 - 9 = 18$$

Solving the two we get a = 6, b = 3 or a = 3, b = 6, which are the required numbers.

Q.2. Th e in ter ior an gles of a polygon ar e in ar ith metic progression. The smallest angle is 120°, and the common difference is 5°, Find the number of sides of the polygon. (1980)

Ans. 9

Sol. Let there be n sides in the polygon.

Then by geometry, sum of all n interior angles of polygon = (n – 2) × 180° Also the angles are in A.P. with the smallest angle = 120° , common difference = 5° ∴ Sum of all interior angles of polygon

$$= \frac{n}{2}[2 \times 120 + (n-1) \times 5]$$

Thus we should have

$$\frac{n}{2}[2\times120+(n-1)\times5]=(n-2)\times180$$

$$\frac{n}{2}[5n+235]=(n-2)\times180$$

$\Rightarrow 5n^2 + 235n= 360n- 720$

$\Rightarrow 5n^2 - 125n + 720= 0$

$\Rightarrow n^2 - 25n + 144= 0$

$\Rightarrow (n -16) (n - 9)= 0$

$\Rightarrow n = 16, \; 9$

Also if n = 16 then 16th angle = 120 + 15 × 5 = 195° > 180°

∴ not possible. Hence n = 9.

Q.3. Does there exist a geometric progression containing 27, 8 and 12 as three of its terms ? If it exits, how many such progressions are possible ? (1982 - 3 Marks)

Ans. yes, infinite

Sol. If possible let for a G.P.

$T_p= 27 = AR^{p-1}$ ....(1)

$T_q= 8 = AR^{q-1}$ ....(2)

$T_r= 12 = AR^{r-1}$ ....(3)

From (1) and (2)

$$R^{p-q}=\frac{27}{8} \Rightarrow R^{p-q}=(3/2)^3 \text{....(4)}$$

From (2) and (3):

$$R^{q-r}=\frac{8}{12} \Rightarrow R^{q-r}=(3/2)^{-1} \text{....(5)}$$

From (4) and (5):

R = 3/2; p – q = 3 ; q – r = – 1 p – 2q + r = 4; p, q, r ∈ N ....(6)

As there can be infinite natural numbers for p, q and r to satisfy equation (6)

∴ There can be infinite G.P's.

Q.4. Find three numbers a, b, c, between 2 and 18 such that (i) their sum is 25 (ii) the numbers 2, a, b sare consecutive terms of an A.P. and (iii) the numbers b, c, 18 are consecutive terms of a G.P. (1983 - 2 Marks)

Ans. a = 5, b = 8, c = 12

Sol. $2 < a, b, c < 18$ $\quad a + b + c = 25$ ....(1)

2, a, b are in AP $\Rightarrow 2a = b + 2 \Rightarrow 2a - b = 2$ ....(2)

b, c, 18 are in GP $\Rightarrow c^2 = 18b$ ....(3)

From (2) $\Rightarrow a = \frac{b+2}{2}$

$$\Rightarrow \frac{b+2}{2} + b + c = 25 \Rightarrow 3b = 48 - 2c$$

$(3) \Rightarrow c^2 = 6(48 - 2c) \Rightarrow c^2 + 12c - 288 = 0$

$\Rightarrow c = 12, -24$ (rejected) $\Rightarrow a = 5, b = 8, c = 12$

Q. 5. If a > 0, b > 0 and c > 0, prove that

$$(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$$ **(1984 - 2 Marks)**

Ans. Sol. Given that a, b, c > 0

We know for +ve numbers A.M. ≥ G.M.

∴ For +ve numbers a, b, c we get

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc} \qquad ....(1)$$

Also for +ve numbers $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, we get

$$\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3} \geq \sqrt[3]{\frac{1}{abc}} \quad ....(2)$$

Multiplying in eqs (1) and (2) we get

$$\left(\frac{a+b+c}{3}\right)\left(\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}\right) \geq \sqrt[3]{abc} \times \frac{1}{\sqrt[3]{abc}}$$

$$\Rightarrow \quad (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9 \text{ Proved.}$$

Q.6. If n is a natural number such that

$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} .... p_k^{\alpha_k}$ **and p_1, p_2, ..., p_k are distinct primes, then show that ln n ≥ k ln2 (1984 - 2 Marks)**

Ans. Sol. Given that $n = p_1^{\alpha_1} . p_2^{\alpha_2} . p_3^{\alpha_3} .... p_k^{\alpha_k}$ ....(1)

Where n∈N and p_1, p_2, p_3, ...p_k are distinct prime numbers.

Taking log on both sides of eq. (1),

we get log n = α_1 log p_1 + α_2 log p_2 + ...+ a_k log p_k ....(2)

Since every prime number is such that

$p_i \geq 2$

$\therefore \quad \log_e p_i \geq \log_e 2$ ....(3)

∀ i= 1 (1) k Using (2) and (3)

we get log n ≥ α_1 log 2 + α_2 log 2 + α_3 log 2 + ...+ α_k log 2

⇒ log n ≥ (α_1+α_2+ ....+ α_k) log 2

⇒ log n ≥ k log 2 Proved.

Q.7. Find the sum of the series :

$\sum_{r=0}^{n}(-1)^r\ {}^nC_r[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}...$ **... up to terms]**

Ans. Sol. The given series is

$$\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\text{.... up to } m \text{ terms}\right]$$

$$\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left[\left(\frac{1}{2}\right)^r+\left(\frac{3}{4}\right)^r+\left(\frac{7}{8}\right)^r+\left(\frac{15}{16}\right)^r+\text{....to } m \text{ terms}\right]$$

Now, $\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(\frac{1}{2}\right)^r=1-{}^nC_1.\frac{1}{2}+{}^nC_2.\frac{1}{2^2}-{}^nC_3.\frac{1}{2^3}+...$

$$=\left(1-\frac{1}{2}\right)^n=\frac{1}{2^n}$$

Similarly, $\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(\frac{3}{4}\right)^r=\left(1-\frac{3}{4}\right)^n=\frac{1}{4^n}$ etc.

Hence the given series is,

$$=\frac{1}{2^n}+\frac{1}{4^n}+\frac{1}{8^n}+\frac{1}{16^n}+\text{....... to terms}$$

$$=\frac{\frac{1}{2^n}\left(1-\left(\frac{1}{2^n}\right)^m\right)}{1-\frac{1}{2^n}}\text{[Summing the G.P.]}$$

$$=\frac{2^{mn}-1}{2^{mn}(2^n-1)}$$

Q.8. Solve for x the following equation : (1987 - 3 Marks)

$\log_{(2x+3)}(6x^2+23x+21)=4-\log_{(3x+7)}(4x^2+12x+9)$

Ans. -1/4

Sol. The given equation is $\log_{(2x+3)}(6x^2+23+21)$

$=4-\log_{3x+7}(4x^2+12x+9)$

$\Rightarrow \log_{(2x+3)} (6x^2 + 23x + 21) + \log_{(3x+7)} (4x^2 + 12x + 9) = 4$

$\Rightarrow \log_{(2x+3)} (2x + 3)(3x + 7) + \log_{(3x+7)} (2x + 3)^2 \; x = 4$

$\Rightarrow 1+\log_{(2x+3)} (3x + 7) + 2\log_{(3x+7)} (2x + 3) = 4$

$\Rightarrow \log_{(2x+3)}(3x + 7) + \frac{2}{\log_{(2x+3)}(3x+7)} = 3$

Let $\log_{(2x+3)}(3x + 7) = y$ ....(1)

$\Rightarrow \; y+\frac{2}{y}=3 \Rightarrow y^2 - 3y+2=0$

$\Rightarrow (y - 1)\;(y - 2) = 0 \Rightarrow y = 1, 2$

Substituting the values of y in (1), we get

$\Rightarrow \log_{(2x+3)}(3x + 7) = 1$ and $\log_{(2x+3)}(3x + 7) = 2$

$\Rightarrow 3x + 7 = 2x + 3$ an d$3x + 7 = (2x + 3)^2$

$\Rightarrow x = - 4$ an d$4x^2 + 9x + 2 = 0$

$\Rightarrow x = - 4$ an d$(x + 2)(4x + 1) = 0$

$\Rightarrow x = - 4$ and $x = - 2$, $x = -¼$

As log a x is defined for $x > 0$ and $a > 0$ $(a \neq 1)$, the possible value of x should satisfy all of the following inequalities :

$\Rightarrow 2x + 3 > 0$ and $3x + 7 > 0$

Also $2x + 3 \neq 1$ and $3x + 7 \neq 1$

Out of $x = - 4$, $x = - 2$ an d $x =$ **–1/4** only $x =$ **–1/4**

satisfies the above inequalities.

So only solution is $x =$ **–1/4**

Q. 9. If $\log 3^{2}$, $\log_3 (2^{x}, 5)$, and $\log_3\left(2^x - \frac{7}{2}\right)$ are in arithmetic progression, determine the value of x. (1990 - 4 Marks)

Ans. 3

Sol. Given that $\log_3 2$, $\log3 (2^x – 5)$, $\log3 (2^x –7/2)$ are in A.P.

$\Rightarrow 2 \log_3 (2^x – 5) = \log_3{}^{2} + \log_3 (2x – 7/2)$

$\Rightarrow (2^{x} - 5)^{2}$

$\Rightarrow (2^x)^2 – 10.2^x + 25 – 2.2^x + 7 = 0 \Rightarrow (2^x)^2 – 12.2^x + 32 = 0$

Let $2^x = y$, then we get, $y^2 – 12y + 32 = 0$

$\Rightarrow (y – 4) (y – 8) = 0$

$\Rightarrow y = 4$ or 8

$\Rightarrow 2x = 22$ or $23 \Rightarrow x = 2$ or 3

But for $\log_3 (2^x – 5)$ and $\log_3 (2^x – 7/2)$ to be defined

$2^x – 5 > 0$ and $2^x – 7/2 > 0 \Rightarrow 2^x > 5$ and $2^x > 7/2$

$\Rightarrow 2^x > 5 \Rightarrow x \neq 2$ and therefore $x = 3$.

Q.10. Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and

$\left(\frac{n+1}{n-1}\right)^2 p$ **.(1991 - 4 Marks)**

Ans. Sol. Let a and b be two numbers and $A_1, A_2, A_3, ... A_n$ be n A.M's between a and b.

Then a, $A_1, A_2, ... A_n$, b are in A.P.

There are (n + 2) terms in the series, so that

$a + (n + 1)d = b \Rightarrow d = \frac{b-a}{n+1}$

$\therefore \quad A_1 = a + \frac{b-a}{n+1} = \frac{an+b}{n+1}$

$\therefore \quad p = \frac{an+b}{n+1}$ ....(1)

The first H.M. between a and b, when nHM's are inserted between a and b can be obtained by replacing a by **1/*a*** and b by **1/*b*** in eq. (1) and then taking its reciptocal.

Therefore, $q = \frac{1}{\frac{\left(\frac{1}{a}\right)n + \frac{1}{b}}{n+1}} = \frac{(n+1)ab}{bn+a}$

$\therefore \quad q = \frac{(n+1)ab}{a+bn}$ ...(2)

We have to prove that q cannot lie between p

$and \ \frac{(n+1)^2}{(n-1)^2}p.$

Now, $n + 1 > n - 1 \Rightarrow \frac{n+1}{n-1} > 1$

$\Rightarrow \left(\frac{n+1}{n-1}\right)^2 > 1$ or $p\left(\frac{n+1}{n-1}\right)^2 > p$

$\Rightarrow \quad p < p\left(\frac{n+1}{n-1}\right)^2$ ....(3)

Now to prove the given, we have to show that q is less than p.

For this, let, $\frac{p}{q} = \frac{(na+b)(nb+a)}{(n+1)^2 ab}$

$\Rightarrow \frac{p}{q} - 1 = \frac{n(a^2+b^2) + ab(n^2+1) - (n+1)^2 ab}{(n+1)^2 ab}$

$$\Rightarrow \frac{p}{q}-1=\frac{n(a^2+b^2-2ab)}{(n+1)^2\,ab}$$

$$\Rightarrow \frac{p}{q}-1=\frac{n}{(n+1)^2}\left(\frac{a-b}{\sqrt{ab}}\right)^2=\frac{n}{(n+1)^2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2$$

$$\Rightarrow \frac{p}{q}-1>0$$

$\Rightarrow$ (provided a and b and hence p and q are +ve) p > q ....(4)

From 3 and (4), we get, $q<p<\left(\frac{n+1}{n+1}\right)^2 p$

$\therefore$ q can not lie between p and $\left(\frac{n+1}{n+1}\right)^2$ p, if a and b are +ve numbers.

Q.11. If S_1, S_2, S_3, ... , S_n are the sums of infinite geometric series whose first terms are 1, 2, 3, ..., n and whose common ratios are

$\frac{1}{2},\frac{1}{3},\frac{1}{4},$ .... $\frac{1}{n+1}$ **respectively,,then find the values**

of $S_1^2+S_2^2+S_3^2+$ .... $+S_{2n-1}^2$ **(1991 - 4 Marks)**

Ans. Sol. We have,

$$S_1=1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+....\infty$$

....

....

$$S_2=2+2.\frac{1}{3}+2\left(\frac{1}{3}\right)^2+....\infty$$

$$S_3=3+3.\frac{1}{4}+3\left(\frac{1}{4}\right)^2+....\infty$$

$$S_n=n+n.\frac{1}{n+1}+n\left(\frac{1}{n+1}\right)^2+....\infty$$

$\Rightarrow \quad S_1 = \frac{1}{1-\frac{1}{2}} = 2 \quad \left[Using\, S_\infty = \frac{a}{1-r}\right]$

$S_2 = \frac{2}{1-\frac{1}{3}} = 3, \quad S_3 = \frac{3}{1-\frac{1}{4}} = 4,$

$S_n = \frac{n}{1-\frac{1}{n+1}} = (n+1)$

$\therefore \quad S_1^2 + S_2^2 + S_3^2 + ... + S_{2n-1}^2$
$= 2^2 + 3^2 + 4^2 + ... + (n+1)^2 + ... + (2n)^2$

NOTE THIS STEP :

$$\sum_{r=1}^{2n} r^2 - 1 = \frac{2n(2n+1)(4n+1)}{6} - 1^2$$

$$= \frac{n(2n+1)(4n+1) - 3}{3}$$

Q.12. The real numbers x_1, x_2, x_3 s atis fying the equation x^3 , $x^2 + \beta x + \gamma < 0$ are in AP. Find the intervals in which β and γ lie. (1996 - 3 Marks)

Ans. Sol. Since x_1, x_2, x_3 are in A.P.

Therefore, let $x_1 = a - d$, $x_2 = a$ and $x_3 = a + d$ and x_1, x_2, x_3 are the roots of $x_3 - x_2 + \beta x + \gamma = 0$

We have $\sum\alpha = a - d + a + a + d = 1$ ....(1)

$\sum\alpha\beta = (a - d)\, a + a\,(a + d) + (a - d)(a + d) = b$ ....(2)

$\alpha\beta\gamma = (a - d)\, a\, (a + d) = -\gamma$ ....(3)

From (1), we get, $3a = 1 \Rightarrow a = 1/3$

From (2), we get, $3a^2 - d^2 = \beta$

$\Rightarrow 3(1/3)^2 - d^2 = \beta \Rightarrow 1/3 - \beta = d^2$

We know that $d^2 > 0\ \forall\ d \in R$

$\frac{1}{3}-\beta \geq 0 \qquad \because d^2 \geq 0$

$\Rightarrow \beta \leq \frac{1}{3} \Rightarrow \beta \in (-\infty, 1/3]$

From (3), $a(a^2 - d^2) = -\gamma$

$\Rightarrow \frac{1}{3}\left(\frac{1}{9}-d^2\right) = -\gamma \Rightarrow \frac{1}{27}-\frac{1}{3}d^2 = -\gamma$

$\Rightarrow \gamma + \frac{1}{27} = \frac{1}{3}d^2 \Rightarrow \gamma + \frac{1}{27} \geq 0$

$\Rightarrow \gamma \geq -\frac{1}{27} \Rightarrow \gamma \in \left[-\frac{1}{27}, \infty\right)$

$\beta \in (-\infty, 1/3)$ and $\gamma \in [-1/27, \infty]$

Q.13. Let a, b, c, d be real numbers in G.P. If u, v, w, satisfy the system of equations (1999 - 10 Marks)

$$u + 2v + 3w = 6$$
$$4u + 5v + 6w = 12$$
$$6u + 9v = 4$$

then show that the roots of the equation

$$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2$$

$$+[(b-c)^2+(c-a)^2+(d-b)^2]\,x+u+v+w=0$$

and $20x^2 + 10(a - d)^2 x - 9 = 0$ are reciprocals of each other.

Sol. Solving the system of equations, $u + 2v + 3w = 6$,

$4u + 5v + 6w = 12$ and $6u + 9v = 4$

we get $u = -1/3$, $v = 2/3$, $w = 5/3$

$\therefore \quad u+v+w=2, \ \frac{1}{u}+\frac{1}{v}+\frac{1}{w}=-\frac{9}{10}$

Let r be the common ratio of the G.P., a, b, c, d. Then b = ar, c = ar^2, d = ar^3 .
Then the first equation

$$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2 + [(b-c)^2 + (c-a)^2 + (d-b)^2]x + (u+v+w) = 0 \text{ becomes}$$

$$-\frac{9}{10}x^2 + [(ar-ar^2)^2 + (ar^2-a)^2 + (ar^3-ar)^2]\,x + 2 = 0$$

i.e., $9x^2 - 10a^2(1-r)^2\,[r^2 + (r+1)^2 + r^2(r+1)^2]x - 20 = 0$
i.e., $9x^2 - 10a^2(1-r)^2\,(r^4 + 2r^3 + 3r^2 + 2r + 1)\,x - 20 = 0$

i.e., $9x^2 - 10a^2(1-r)^2\,(1 + r + r^2)^2\,x - 20 = 0,$
i.e., $9x^2 - 10a^2(1-r^3)^2\,x - 20 = 0$ ....(1)

The second equation is, $20x^2 + 10(a - ar^3)^2\, x - 9 = 0$

i.e., $20x^2 + 10a^2(1-r^3)^2 x - 9 = 0$ ....(2)

Since (2) can be obtained by the substitution $x \to 1/x$, equations (1) and (2) have reciprocal roots.

Q.14. The fourth power of the common difference of an arithmatic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. (2000 - 4 Marks)

Ans.

Sol. Let a – 3d, a – d, a + d and a + 3d be any four consecutive terms of an A.P. with common difference 2d.

∵ Terms of A.P. are integers, 2d is also an integer.

Hence $P = (2d)^4 + (a-3d)(a-d)(a+d)(a+3d)$

$= 16d^4 + (a^2 - 9d^2)(a^2 - d^2) = (a^2 - 5d^2)^2$

Now, $a^2 - 5d^2 = a^2 - 9d^2 + 4d^2 = (a-3d)(a+3d) + (2d)^2 =$ some integer

Thus, P = square of an integer.

Q.15. Let $a_1, a_2, \ldots, a_n$ be positive real numbers in geometric progression. For each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of $a_1, a_2, \ldots, a_n$. Find an expression for the geometric mean of $G_1, G_2, \ldots, G_n$ in terms of $A_1, A_2, \ldots, A_n, H_1, H_2, \ldots, H_n$. (2001 - 5 Marks)

Ans.

Sol. Given that $a_1, a_2, \ldots a_n$ are +ve real no's in G.P.

$$\therefore \left.\begin{array}{l} a_1 = a \\ a_2 = ar \\ a_3 = ar^2 \\ \vdots \\ a_n = ar^{n-1} \end{array}\right\} \text{As } a_1, a_2, \ldots a_n \text{ are +ve} \quad \therefore r>0$$

A_n is A.M. of $a_1, a_2, \ldots, a_n$

$$\therefore \quad A_n = \frac{a_1 + a_2 + \ldots + a_n}{n} = \frac{a + ar + \ldots + ar^{n-1}}{n}$$

$$A_n = \frac{a(1-r^n)}{n(1-r)} \ \ldots (1) \quad (\text{For } r \neq 1)$$

G_n is G.M. of $a_1, a_2, \ldots, a_n$

$$\therefore \quad G_n = \sqrt[n]{a_1, a_2 \ldots a_n} = \sqrt[n]{a.ar.ar^2 \ldots ar^{n-1}}$$

$$= n\sqrt{a^n, r\frac{n(n-1)}{2}} = ar^{\frac{(n-1)}{2}}$$

$$G_n = ar^{\frac{(n-1)}{2}} \quad \ldots (2) \quad (r \neq 1)$$

Q.15. Let $a_1, a_2, \ldots, a_n$ be positive real numbers in geometric progression. For each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of $a_1, a_2, \ldots, a_n$. Find an expression for the geometric mean of $G_1, G_2, \ldots, G_n$ in terms of $A_1, A_2, \ldots, A_n, H_1, H_2, \ldots, H_n$. (2001 - 5 Marks)

Sol. Given that $a_1, a_2, \ldots a_n$ are +ve real no's in G.P.

$$\therefore \left.\begin{array}{l} a_1 = a \\ a_2 = ar \\ a_3 = ar^2 \\ \vdots \\ a_n = ar^{n-1} \end{array}\right\} \text{As } a_1, a_2, \ldots a_n \text{ are +ve} \quad \therefore r>0$$

A_n is A.M. of $a_1, a_2, \ldots, a_n$

$$\therefore \quad A_n = \frac{a_1 + a_2 + \ldots + a_n}{n} = \frac{a + ar + \ldots + ar^{n-1}}{n}$$

$$A_n = \frac{a(1-r^n)}{n(1-r)} \ldots (1) \quad (\text{For } r \neq 1)$$

G_n is G.M. of $a_1, a_2, \ldots, a_n$

$$\therefore \quad G_n = \sqrt[n]{a_1, a_2 \ldots a_n} = \sqrt[n]{a.ar.ar^2 \ldots ar^{n-1}}$$

$$= n\sqrt{a^n, r\frac{n(n-1)}{2}} = ar^{\frac{(n-1)}{2}}$$

$$G_n = ar^{\frac{(n-1)}{2}} \qquad \ldots (2) \quad (r \neq 1)$$

H_n is H.M. of $a_1, a_2, \ldots, a_n$

$$\therefore \quad H_n = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}} = \frac{n}{\frac{1}{a} + \frac{1}{ar} + \ldots + \frac{1}{ar^{n-1}}}$$

$$= \frac{n}{\frac{1}{a}\frac{\left(\frac{1}{r^n} - 1\right)}{\left(\frac{1}{r} - 1\right)}} = \frac{n}{\frac{1}{a}\left(\frac{1-r^n}{r^n}\right) \cdot \frac{r}{1-r}}$$

$$H_n = \frac{anr^{n-1}(1-r)}{(1-r^n)} \ (r \neq 1) \ldots (3)$$

We also observe that

$$A_n H_n = \frac{a(1-r^n)}{(n(1-r))} \times \frac{anr^{n-1}(1-r)}{(1-r^n)} = a^n r^{n-1} = G_n^2$$

$\therefore A_n H_n = G_n^2 \ldots (4)$

$\therefore$ Now, G.M. of $G_1, G_2, \ldots G_n$ is

$$G = {}^n\sqrt{G_1 \ G_2 \ldots G_n}$$

$G=\sqrt[n]{\sqrt{A_1H_1}\ \sqrt{A_2H_2}\ldots\sqrt{A_nH_n}}$ [Using equation (4)

$G=(A_1A_2....A_nH_1H_2....H_n)1/2n$ ...(5)

If $r=1$ then $A_n=G_n=H_n=a$

Also $A_n\ H_n=G_n{}^2$

∴ For r = 1 also, equation (5) holds.

Hence we get, $G=(A_1A_2....A_nH_1H_2....H_n)^{1/2n}$

Q.16. Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression,

show that $\frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}=\frac{(2a+b)(a+2b)}{9ab}$. **(2002 - 5 Marks)**

Ans.

Sol. Clearly $A_1+A_2=a+b$

$$\frac{1}{H_1}+\frac{1}{H_2}=\frac{1}{a}+\frac{1}{b}$$

$$\Rightarrow \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2} \Rightarrow \frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$$

Also $\frac{1}{H_1}=\frac{1}{a}+\frac{1}{3}\left(\frac{1}{b}-\frac{1}{a}\right) \Rightarrow H_1=\frac{3ab}{2b+a}$

$$\frac{1}{H_2}=\frac{1}{a}+\frac{2}{3}\left(\frac{1}{b}-\frac{1}{a}\right) \Rightarrow H_2=\frac{3ab}{2a+b}$$

$$\Rightarrow \frac{A_1+A_2}{H_1+H_2}=\frac{a+b}{3ab\left(\frac{1}{2b+a}+\frac{1}{2a+b}\right)}$$

$$=\frac{(2b+a)(2a+b)}{9ab}$$

Q.17. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either a = b = c or a, b, form a G.P.. (2003 - 4 Marks)

Ans.

Sol. Given that a, b, c are in A.P.

$\Rightarrow$ 2b = a + c ....(1)

and a^2, b^2, c^2 are in H.P.

$$\Rightarrow \frac{1}{b^2}-\frac{1}{a^2}=\frac{1}{c^2}-\frac{1}{b^2}$$

$$\Rightarrow \frac{(a-b)(a+b)}{b^2a^2}=\frac{(b-c)(b+c)}{b^2c^2}$$

$\Rightarrow ac^2 + bc^2 = a^2b + a^2c \quad [\because a - b = b - c]$

$\Rightarrow$ ac (c – a) + b (c – a) (c + a) = 0

$\Rightarrow$ (c – a) (ab + bc + ca) = 0

$\Rightarrow$ either c – a = 0 or ab + bc + ca = 0

$\Rightarrow$ either c = a or (a + c) b + ca = 0 and then from (i)

$2b^2 + ca = 0$

Either a = b = c or $b^2 = a\left(\frac{-c}{2}\right)$

i.e. a, b, – c/2 are in G.P. Hence Proved.

Q.18. If $a_n=\frac{3}{4}-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+.....(-1)^{n-1}\left(\frac{3}{4}\right)^n$ and

b_n = 1 – a_n, then find the least natural number n_0 such that

$b_n > a_n \ \forall n \geq n_0$. **(2006 - 6M)**

Ans.

Sol. $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$

$$= \frac{\frac{3}{4}\left(1 - \left(-\frac{3}{4}\right)^n\right)}{1 + \frac{3}{4}} = \frac{3}{7}\left(1 - \left(-\frac{3}{4}\right)^n\right)$$

$b_n = 1 - a_n$ and $b_n > a_n \ \forall \ n \geq n_0$

$\therefore \ 1 - a_n > a_n \Rightarrow 2a_n < 1$

$$\Rightarrow \frac{6}{7}\left[1 - \left(-\frac{3}{4}\right)^n\right] < 1 \Rightarrow -\left(-\frac{3}{4}\right)^n < \frac{1}{6}$$

$\Rightarrow (-3)^{n+1} < 2^{2n-1}$ For n to be even, inequality always holds. For n to be odd, it holds for $n \geq 7$.

$\therefore$ The least natural no., for which it holds is 6

($\because$ it holds for every even natural no.)

Integer Type ques of Sequences and Series

Q.1. Let S_k, k = 1, 2, ... , 100, denote the sum of the infinite geometric series whose first term is

$\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of

$\frac{100^2}{100!}+\sum_{k=1}^{100}|(k^2-3k+1)S_k|$ is **(2010)**

Ans. (3)

Sol. Using $S_\infty=\frac{a}{1-r}$, we get

$$S_k=\begin{cases}\dfrac{\frac{k-1}{k!}}{1-\frac{1}{k!}}, & k\neq 1\\ 0, & k=1\\ \dfrac{1}{(k-1)!}, & k\geq 2\end{cases}$$

Now $\sum_{k=1}^{100}|(k^2-3k+1)S_k|=\sum_{k=2}^{100}|(k^2-3k+1)|\frac{1}{(k-1)!}$

$$=|-1|+\sum_{k=3}^{100}\frac{(k^2-1)+1-3(k-1)-2}{(k-1)!}\quad as\ k^2-3k+1>0\ \forall k\geq 3$$

$$=1+\sum_{k=3}^{100}\left(\frac{1}{(k-3)!}-\frac{1}{(k-1)!}\right)$$

$$=1+\left(1-\frac{1}{2!}\right)+\left(\frac{1}{1!}-\frac{1}{3!}\right)+\left(\frac{1}{2!}-\frac{1}{4!}\right)+....$$

$$....+\left(\frac{1}{96!}-\frac{1}{98!}\right)+\left(\frac{1}{97!}-\frac{1}{99!}\right)$$

$$= 3 - \frac{1}{98!} - \frac{1}{99!} = 3 - \frac{9900}{100!} - \frac{100}{100!} = 3 - \frac{10000}{100!} = 3 - \frac{(100)^2}{100!}$$

$$\therefore \frac{100^2}{100!} + 3\sum_{k=1}^{100}\left|\left(k^2 - 3k + 1\right)S_k\right| = 3.$$

Q.2. Let $a_1, a_2, a_3 ...., a_{11}$ be real numbers satisfying

$a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, ...11$.

if $\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90$ **, then the value of**

$\frac{a_1 + a_2 + ... + a_{11}}{11}$ **is equal to (2010)**

Sol. Given that $a_k = 2a_{k-1} - a_{k-2}$

$$\Rightarrow \frac{a_{k-2} + a_k}{2} = a_{k-1}, 3 \le k \le 11$$

$\Rightarrow a_1, a_2, a_3, ..., a_{11}$ are in AP..

If a is the first term and D the commen difference then

$a^2_1 + a^2_2 + ... + a^2_{11} = 990$ (

$\Rightarrow 11a^2 + d^2 (1^2 + 2^2 + ... + 10^2) + 2ad (1 + 2 + ... + 10) = 990$

$$\Rightarrow 11a^2 + \frac{10 \times 11 \times 21}{6} d^2 + 2ad \times \frac{10 \times 11}{2} = 990$$

$\Rightarrow a^2 + 35d^2 + 150d = 90$ Using $a = 15$,

we get $35d^2 + 150d + 135 = 0$ or $7d^2 + 30d + 27 = 0$

$\Rightarrow d + 3)(7d + 9) = 0 \Rightarrow d = -3$ or $-9/7$

then $a^2 = 15 - 3 = 12$ or $15 - \frac{9}{7} = \frac{96}{7} > \frac{27}{2}$

$\therefore d \ne -97$

Hence $\frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{\frac{11}{2}\left[2 \times 15 + 10(-3)\right]}{11} = 0$

Q.3. Let a_1, a_2, a_3 .....a100 be an arithmetic progression with $a_1 = 3$

and $a_1 = 3$ **and** $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100.$ **For any integer n with $1 \le n \le 20$, let m = 5n.**

If $\frac{S_m}{S_n}$ **does not depend on n, then a_2 is (2011)**

Ans. (9)

Sol. We have $\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[2 \times 3 + (5n-1)d]}{\frac{n}{2}[6 + (n-1)d]}$

$= \frac{5[(6-d) + 5nd]}{(6-d) + nd]}$

which will be independent of n if d = 6 or d = 0 For a proper A.P. we take d = 6 then $a_2 = 3 + 6 = 9$

Q.4. A pack con tain s n cards n umber ed fr om 1 to n . Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 = (JEE Adv. 2013)

Ans. (5)

Sol. Let k, k + 1 be removed from pack.

$\therefore (1 + 2 + 3 + ... + n) - (k + k + 1) = 1224$

$\frac{n(n+1)}{2} - 2k = 1225$

$k = \frac{n(n+1) - 2450}{4}$

for n = 50, k = 25 $\therefore$ k – 20 = 5

Q.5. Let a, b, c be positive integers such that b/a is an integer. If a,b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is (JEE Adv. 2014)

Ans. (4)

Sol. $\because$ a, b, c are in G.P

$\therefore$ b = ar and c = ar^2

Also $\frac{b}{a}$ is an integer

$\Rightarrow$ r is an integer

$\because$ A.M. of a, b, c is b + 2

$\Rightarrow \frac{a+b+c}{3} = b+2$

$\Rightarrow a + ar + ar^2 = 3ar+6$

$\Rightarrow a(r^2 - 2r + 1)=6$

$\Rightarrow a(r - 1)^2=6$.

$\because$ a and r are integers

$\therefore$ The only possible values of a and r can be 6 and 2 respectively.

Then $\frac{a^2+a-14}{a+1} = \frac{36+6-14}{6+1} = \frac{28}{7} = 4$

Q.6. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is (JEE Adv. 2015)

Sol.

$$\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11} \Rightarrow a = 9d$$

$a_7 = a + 6d = 15d$

$\because$ $130 < 15d < 140 \Rightarrow d = 9$

($\because$ All terms are natural numbers $\therefore$ $d \in N$)

Q.7. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3) \dots (1 + x^{100})$ is (JEE Adv. 2015)

Ans. (8)

Sol. In expansion of $(1 + x)(1 + x^2)(1 + x^3) \dots (1 + x^{100})$ x^9 can be found in the following ways x^9, x^{1+2} 8, x^{2+7}, x^{3+6}, x^{4+5}, x^{1+2+6}, x^{1+3+5}, x^{2+3+4} The coefficient of x9 in each of the above 8 cases is 1. $\therefore$ Required coefficient = 8.